

Úloha 3

Zjednodušte

$$\frac{(x + x^{-1})^{-3} + (x - x^{-1})^{-3}}{(x^2 - x^{-2})^{-3}} =$$

(Adamův a Jankův příklad, rok 2017)

Úloha 3

Zjednodušte

$$\frac{(x+x^{-7})^{-3} + (x-x^{-7})^{-3}}{(x^2-x^{-2})^{-3}} =$$

(Adamův a Jankův příklad, rok 2017)

Řešení

$$\begin{aligned} & \frac{(x+x^{-7})^{-3} + (x-x^{-7})^{-3}}{(x^2-x^{-2})^{-3}} = && x \neq 0 \\ & && x \neq 1 \\ & && x \neq -1 \\ & = \frac{\left(x + \frac{1}{x}\right)^{-3} + \left(x - \frac{1}{x}\right)^{-3}}{\left(x^2 - \frac{1}{x^2}\right)^{-3}} = \\ & = \frac{\left(\frac{x^2+1}{x}\right)^{-3} + \left(\frac{x^2-1}{x}\right)^{-3}}{\left(\frac{x^4-1}{x^2}\right)^{-3}} = \\ & = \frac{\left(\frac{x}{x^2+1}\right)^3 + \left(\frac{x}{x^2-1}\right)^3}{\left(\frac{x^2}{x^4-1}\right)^3} = \\ & = \frac{\left(\frac{x}{x^2+1}\right)^3}{\left(\frac{x^2}{x^4-1}\right)^3} + \frac{\left(\frac{x}{x^2-1}\right)^3}{\left(\frac{x^2}{x^4-1}\right)^3} = \frac{(x^2-1)(x^2+1) \cdot x}{x^2(x^2+1)} + \\ & \quad + \frac{x(x^2-1)(x^2+1)}{x^2(x^2-1)} = \\ & = \frac{x^2-1}{x} + \frac{x^2+1}{x} = \\ & = \frac{(x^2-1)(x^2+1)}{x} = \frac{x^4-1}{x} \end{aligned}$$